Exercise 1.42

Given two vectors $\vec{\boldsymbol{A}} = 4.00\hat{\boldsymbol{i}} + 7.00\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{B}} = 5.00\hat{\boldsymbol{i}} - 2.00\hat{\boldsymbol{j}}$, (a) find the magnitude of each vector; (b) write an expression for the vector difference $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$ using unit vectors; and (c) find the magnitude and direction of the vector difference $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$. (d) In a vector diagram show $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$, and also show that your diagram agrees qualitatively with your answer in part (c).

Solution

The magnitudes of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ are

$$\begin{aligned} |\vec{\boldsymbol{A}}| &= \sqrt{(4.00)^2 + (7.00)^2} \approx 8.06 \\ |\vec{\boldsymbol{B}}| &= \sqrt{(5.00)^2 + (-2.00)^2} \approx 5.39. \end{aligned}$$

The vector difference is

$$\begin{aligned} \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} &= (4.00\hat{\boldsymbol{i}} + 7.00\hat{\boldsymbol{j}}) - (5.00\hat{\boldsymbol{i}} - 2.00\hat{\boldsymbol{j}}) \\ &= (4.00 - 5.00)\hat{\boldsymbol{i}} + (7.00 + 2.00)\hat{\boldsymbol{j}} \\ &= -1.00\hat{\boldsymbol{i}} + 9.00\hat{\boldsymbol{j}}, \end{aligned}$$

and its magnitude and direction measured counterclockwise from the positive x-axis are

$$\begin{aligned} |\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}| &= \sqrt{(-1.00)^2 + (9.00)^2} \approx 9.06 \\ \theta &= \tan^{-1}\left(\frac{9.00}{-1.00}\right) \\ &= \pi - \tan^{-1}\left(\frac{9.00}{1.00}\right) \\ &\approx 96.3^\circ. \end{aligned}$$